Geach's Deontic Quantifier

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Abstract

We show that Peter Geach's ideas on deontic quantification or deontic abstraction (*Philosophia* 11 (1981) 1–12) pose no threat to the standard modal approach towards deontic logic. They can very well be represented within the framework of a standard quantified deontic system equipped with an alethic abstraction operator.

In 1981 Peter Geach published an article in which he criticized the standard, modal approach towards deontic logic for representing deontic notions by means of *propositional operators*.[1] In this approach a sentence like "we ought to pay" is analyzed as "(it ought to be the case that)(we pay)". Geach argued that it would be better to represent deontic notions by means of complex *predicates*. The just-mentioned sentence should accordingly be analyzed as "(ought to pay)(we)", where "(ought to pay)" is a predicate. Geach argued that this analysis has two advantages over propositional modal deontic logic: (1) it is in better accord with ordinary language; and (2) it enables us to make finer distinctions than the standard approach does. For example, as early as the eleventh century St. Anselm knew that "John ought to beat up Tom" is not necessarily equivalent with "Tom ought to be beaten up by John". Geach argued that these sentences are logically equivalent according to propositional modal deontic logic because "John beats up Tom" and "Tom is beaten up by John" cannot fail to express the same proposition (describe the same possible state of affairs).

Geach proposed the following alternative analysis. Take a sentence such as $Pn \wedge Qn$, where P and Q represent certain verbs. This sentence may be analyzed as the result of attaching a predicate $\lambda x(Px \wedge Qx)$ to a name n, where λ is the usual abstraction operator. Thus, the sentence is of the form:

$$(\lambda x(Px \wedge Qx))n$$

To say that n *ought to* perform rather than *does* perform the kind of action represented by $\lambda x(Px \wedge Qx)$, we write:

$$(Ox(Px \wedge Qx))n$$

[1]Peter T. Geach, "Whatever happened to deontic logic?" *Philosophia* 11 (1981) 1–12. Reprinted in Peter T. Geach, ed., *Logic and Ethics*, Dordrecht etc., Kluwer Academic Publishers, 1991, pp. 33–48. Geach's article is strongly reminiscent of Timothy C. Potts, "Modal logic and auxiliary verbs," in Carl H. Heidrich, ed., *Semantics and Communication*, Amsterdam etc., North-Holland Publishing Company, 1974, pp. 180–209. Although Geach and Potts were both working at the University of Leeds they did not mention each other. The standard approach towards deontic logic is described in Lennart Åqvist, "Deontic logic," in Dov M. Gabbay and Franz Günthner, eds., *Handbook of Philosophical Logic, Vol. II: Extensions of Classical Logic*, Dordrecht, D. Reidel, 1984, pp. 605–714.

Here O is a *deontic abstraction operator* or *deontic quantifier* which is syntactically analogous to λ.[2] $Ox(Px \wedge Qx)$ is a predicate which may be read as "ought to perform P and Q".

To give another example, consider the sentence "n ought (to do P if he ought to do P)". First, we symbolize the sentence "n does P if he ought to do P":

$$(OxPx)n \rightarrow Pn$$

The corresponding predicate is:

$$\lambda y((OxPx)y \rightarrow Py)$$

To say that this *ought* to be the way n acts we now write:

$$(Oy((OxPx)y \rightarrow Py))n \tag{1}$$

It is not entirely clear whether Geach viewed his proposal as an addition to standard deontic logic or as an alternative to it. His appeal to a certain well-known principle from deontic logic in order to derive $(Ox(Px \rightarrow Qx))n$ from $(Ox\neg Px)n$ gives the former impression, his diatribe against the usual "deontically perfect possible worlds" semantics of the standard approach the latter. Geach was so vague about the logical properties of his quantifier that one cannot decide. In his recent discussion of Geach's proposal,[3] Hilpinen did nothing to clarify this issue.

In the present note, we shall bring some clarity where Geach left us in the dark. We shall show that it is quite possible to define Geach's deontic quantifier within the standard modal approach.

In order to keep matters simple, we shall define our deontic system DML in terms of another system, Bencivenga's and Woodruff's alethic modal system ML.[4] ML need not be described in detail. Suffice it to say: (1) that it has an abstraction operator λ, which is such that if A is a formula, $\lambda x_1 \ldots\ x_n A$ is an n-ary predicate, and (2) that λ has the following semantical interpretation:

$$I_w(\lambda x_1 \ldots\ x_n A) = \{\langle d_1, \ldots, d_n\rangle \in D^n : I_w(A[d_1^*/x_1, \ldots, d_n^*/x_n]) = T\},$$

where I_w is the interpretation at possible world w, D is the domain, T is truth, and each d_i^* is an individual constant such that $I_v(d_i^*) = d_i$ for all possible worlds v. In other words, $\lambda x_1 \ldots\ x_n A$ is, just like any other n-ary predicate, interpreted as a set of ordered n-tuples of objects in the domain.

ML can be transformed into deontic system DML by means of an inverse application of the familiar "Anderson-reduction".[5] That is, the language is

[2]Geach's notation was slightly different from ours. He wrote $(Px \wedge Qx)n$ and $(Ox)n(Px \wedge Qx)$ instead of $(\lambda x(Px \wedge Qx))n$ and $(Ox(Px \wedge Qx))n$. Geach's notation makes it less clear that O is an abstraction operator which is syntactically similar to λ. We owe the term "deontic quantifier" to Risto Hilpinen, "Actions in deontic logic," in John-Jules Ch. Meyer and Roel J. Wieringa, eds., *Deontic Logic in Computer Science*, Chichester, John Wiley and Sons, 1993, pp. 85–100. This term is not entirely fortunate because quantifiers transform formulas into formulas whereas λ and O transform formulas into predicates.

[3]See the previous note.

[4]Ermanno Bencivenga and Peter W. Woodruff, "A new modal language with the λ operator," *Studia Logica* 40 (1981) 383–389. See also Robert C. Stalnaker and Richmond H. Thomason, "Abstraction in first-order modal logic," *Theoria* 34 (1968) 203–207.

[5]Alan Ross Anderson, "A reduction of deontic logic to alethic modal logic," *Mind* 67 (1958) 100–103.

supplemented with a propositional constant V (usually read as "all hell breaks loose") and an obligation operator $\boldsymbol{O}$ which is defined by:

$$\boldsymbol{O}A \stackrel{\text{def}}{=} \Box(\neg A \to V),$$

where $\Box$ is the necessity operator from ML. (The obligation operator is written in boldface in order to distinguish it from the deontic quantifier.)

We may now give the following definition of Geach's quantifier:

$$Ox_1 \dots\ x_n A \stackrel{\text{def}}{=} \lambda x_1 \dots\ x_n \boldsymbol{O} A \qquad (2)$$

Thus (1), for example, turns out to be an abbreviation of:

$$(\lambda y \boldsymbol{O}((\lambda x \boldsymbol{O} Px)y \to Py))n$$

It will be clear that DML is just an example of a system in which Geach's quantifier can be defined. (2) is expressible in any deontic system which contains the λ operator.

We may now make the following observations. Geach could not make them because he did not give a clear semantic or axiomatic characterization of his deontic quantifier.

1. Geach wrote that "it looks as though we may pass from $(Ox\neg Px)n$ to $(Ox(Px \to Qx))n$." We need not be so cautious: this inference is correct according to DML.

2. DML vindicates St. Anselm's observation about the non-equivalence of deontic assertions involving the passive and active voices. The formulas $(OxPxm)n$ ("n ought to beat up m") and $(OxPnx)m$ ("m ought to be beaten up by n") are indeed not logically equivalent.

3. Formula (1) ("n ought (to do P if he ought to do P)") is in general not equivalent with the following formula ("n ought (to do P if n ought to do P)"):

$$(Oy((OxPx)n \to Py))n \qquad (3)$$

   Similar distinctions are well-known from the fields of epistemic and perceptual logic.[6] It is a sad fact that they are usually ignored in the literature about standard deontic logic with agent-relativized operators (in which $\boldsymbol{O}_i A$, for example, stands for "agent i is obliged to do A").

4. Is $(OxA)n \equiv \boldsymbol{O}A[n/x]$ valid or not? The answer does not matter. The important thing to notice is that Geach could not even consider such "bridge laws" between standard deontic logic and his own system (of which (2) is also an example) because he could not express them in his language.

We conclude that Geach's ideas on deontic quantification or deontic abstraction pose no threat to the standard modal approach towards deontic logic. They can very well be represented within the framework of a standard quantified deontic system equipped with an alethic abstraction operator.

[6] See, for example, Romane Clark, "Old foundations for a logic of perception," *Synthese* 33 (1976) 75–99.